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An Evaluation of Property Tax Regressivity in Philadelphia (2013 – 2017)

KEY FINDINGS

- Property assessments in the combined city and county of Philadelphia are moderately regressive, with levels comparable to or below many other metros evaluated for this series.
- The lowest-valued properties in Philadelphia received average assessments equal to 130% of their sale price, while the city's highest-valued properties were assessed at less than 90% of their sale price.
- As a result, more than $165 million in property value goes untaxed every year, among recently sold homes alone.

INTRODUCTION

The property tax is the single largest source of revenue for American local governments. Cities, counties, school districts, and special districts raise roughly $500 billion per year in property taxes, accounting for 72% of local taxes and 47% of local own-source general revenue, nationwide.[1] Whether residents rent or own, property taxes directly or indirectly impact almost everyone.

In many cities, however, property taxes are inequitable: low-value properties face higher tax assessments, relative to their actual sale price, than do high-value properties, resulting in regressive taxation that burdens low-income residents disproportionately. The Center for Municipal Finance at the University of Chicago has evaluated the regressivity of property assessment in 14 of America's largest cities and counties. The following report highlights the system in the combined city-county of Philadelphia, where property taxes account for roughly 14% of all own-source revenue.[2]

[1] *U.S. Census Bureau, 2016 Annual Surveys of State and Local Government Finances.* https://www.census.gov/data/datasets/2016/econ/local/public-use-datasets.html.
[2] *Annual Survey of State and Local Government Finances*, United States Senate (last accessed October 2017), https://census.gov/programs-surveys/gov-finances.html.

Among the fourteen communities evaluated for this series, Philadelphia demonstrated moderate to modest levels of regressivity. Moreover, this regressivity has improved substantially in recent years, though levels of the most common industry measures of regressivity remain in excess of acceptable limits. On average, the city's lowest-valued homes are assessed at more than 132% of their market value, while the highest-valued homes are assessed at only 87% of their market value. This disparity produces significant tax consequences as well, leaving more than $165 million in untaxed property value among recently-sold homes, each year. Similarly, all three of the most common methods for evaluating assessment equity indicate regressive results for Philadelphia. The city has, however, been trending closer toward acceptable norms: all three of the industry measures of assessment accuracy and regressivity appear far beyond acceptable limits at the beginning of our observation period in 2013 but have either come within acceptable limits, or nearly so, since then.

Unlike most metro communities, the city of Philadelphia operates as a combined city-county government. As such, assessments are conducted county-wide, and thus, the following evaluation focuses on properties throughout the entire combined city-county. The report at hand relies on data provided by the Philadelphia Assessor and covers all "arms-length" property transactions within the city between 2013 and 2017. The analyses that follow use only "arms-length" transactions, generally meaning only traditional, market-rate sales involving buyers and sellers with no previous relationship (rather than, for example, sales between relatives or foreclosure auctions). For these analyses, we use the local assessor's classification of arms-length transactions.[3]

[3] For an explanation and example of how the measures used in this paper may vary depending on local versus IAAO definitions of "arms-length" see the Center's previous work regarding St. Louis and St. Louis County assessments, which can be found at www.propertytaxproject.uchicago.edu/papers.

The standard approach for evaluating the quality and fairness of assessments is through a sales ratio study.[4] The *sales ratio* is defined as the assessed value of a property divided by its sale price. A sales ratio study evaluates the extent of regressivity in a jurisdiction, along with other aspects of assessment performance, by studying sales ratios for properties that sold within a specific time period. A system in which less expensive homes are systematically assessed at higher sales ratios than more expensive homes is *regressive*.

This report presents a basic sales ratio study for the city of Boston based on data provided by the local assessor's office. Following a conceptual review of regressivity, our findings are broken into three categories: 1) the results of our sales-ratio study, 2) the application of industry standard measures of regressivity, and 3) the tax implications of local regressivity and inaccuracy.

Understanding Assessment Regressivity and Its Consequences

The property tax is, in principle, an *ad valorem* tax, meaning that the tax is proportional to the value of the property. Most textbook discussions of the property tax proceed as though a property's value is well known. But this is seldom the case. For a property that has sold recently, the sale price is usually a reasonable approximation of its market value. But only a small proportion of properties change hands in any given year— roughly 3-9% of all homes each year according to our data. For the vast majority of properties, which have not sold recently, the value must somehow be estimated. This is the job of local assessors.

In most large jurisdictions, assessors rely on statistical models to assess residential property. This procedure is, essentially, as follows:

- The local assessor compiles a list of all of the properties which have sold recently and identifies important characteristics of each property such as square footage, the number of bedrooms, the size of the yard, the age of the property, etc.

[4] *See* International Association of Assessing Officers. 2013. *Standard on Ratio Studies.* https://www.iaao.org/media/standards/Standard_on_Ratio_Studies.pdf.

- The assessor estimates the relationship between a property's features and its' market value, using data from the sample of recently sold properties. For instance, each additional square foot of building space adds some amount to the sale price, an additional bathroom adds a certain amount of value, and so on. A statistical model, such as a regression, is created to estimate the relationships between all potentially relevant property features and the sale price.
- This statistical model is used to estimate the values of all similarly situated homes that haven't sold, based on their features. That is, the assessor assumes that the relationship between property features and prices for the sold properties would have been the same for the unsold properties. For example, if, among properties that sold, the average price for a 2,000 square foot, 3-bedroom home was $100,000, the assessor assumes that other 2,000 square foot, 3-bedroom homes that weren't sold are worth $100,000. In principle, these comparisons should be limited to homes within the same neighborhood, since the price of similar homes can vary significantly across locations, particularly in larger communities.
- The assessed value from this process becomes the basis on which property taxes are levied. Various exemptions and deductions may be applied at this stage.
- These assessments may be adjusted after the fact as the result of appeals by property owners.

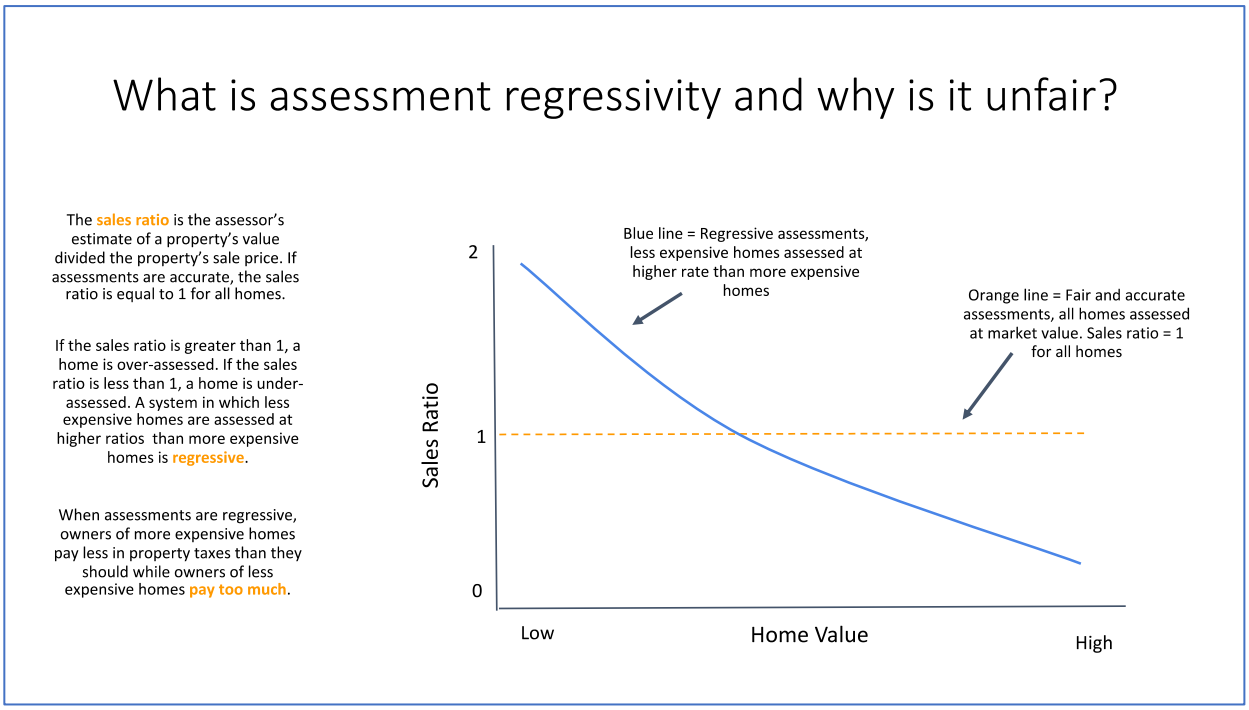
When assessment is conducted accurately, the resulting property taxes indeed constitute an *ad valorem* tax. However, when property assessment is inaccurate, the resulting property taxes will also be inaccurate. Over-assessed properties will be over-taxed, while under-assessed properties will be under-taxed. Although no assessment system is perfectly accurate, we are especially concerned with a particular type of inaccuracy known as *regressivity*. Assessments are regressive when low-value homes are assessed at a higher percentage of their true market value than are high-value homes.

To understand regressive assessment and its consequences, it is useful to contrast it with fair assessment. A common way of diagnosing regressivity is to compare the *sales ratio* for homes with different sale prices.[5]

Figure 1 shows what the average sales ratio should look like in a properly functioning assessment system, as well as what can go wrong when assessments are regressive. If assessments were perfectly accurate, every home would be valued at exactly 100% of its value, meaning that the sales ratio would be 1 for every property, as depicted by the dashed orange line. Of course, no assessment system if perfect. But if the average sales ratio is equal across the spectrum of prices, even an imperfect system will be unbiased with respect to price, meaning that owners of both more and less expensive property will pay their fair share of taxes on average. However, when the average sales ratio is higher for low-priced homes than for high-priced homes, as depicted by the solid blue line, assessments are regressive. Regressive assessments lead to regressive taxation, in which owners of low-value property pay too much in taxes while owners of high-value properties pay too little.

[5] Because accurate sale prices are only known for properties that have recently sold, the sales ratio can only be computed for properties that have recently sold.

Figure 1: Understanding Assessment Regressivity



A simple numerical example illustrates the consequences of assessment regressivity. Suppose the average home that sold for $100,000 is actually assessed at $120,000. Meanwhile, the average home that sold for $1 million is assessed at $800,000. Suppose, the statutory tax rate is 1% of assessed value. In this scenario, the $100,000 home pays $1,200 in taxes each year, for an effective tax rate of 1.2 percent. The $1 million home pays $8,000 in taxes, for an effective tax rate 0.8 percent. The result is that the low-priced home has a 50% higher tax rate than the high-priced home (1.2/0.8 = 1.5).

Graphs such as the one shown in Figure 1 are a useful way to visually detect assessment regressivity. For more formal evaluations, the industry has developed several statistical tests for assessment regressivity. As discussed below, the measures most commonly used by professional assessors are the coefficient of dispersion (COD), price-related differential (PRD) and the coefficient of price-related bias (PRB). In addition, academic researchers have

developed several more sophisticated statistical tests for assessment regressivity.[6] While none of these tests is perfect, collectively they can be used to evaluate the likely extent of assessment regressivity in a given jurisdiction.

The International Association of Assessing Officers (IAAO) provides standards for assessments including standards for uniformity and regressivity (*aka* vertical equity). *Uniformity* refers to the overall level of variability in sales ratios across properties. Regressivity refers to the correlation between sales ratios and sale prices. The three main standards are[7]:

- Coefficient of Dispersion (COD) is a measure of uniformity based on the average percentage deviation of the ratios from the median, expressed as a percentage of the median. For example, given a COD of 15%, a property worth $100,000 has a 50% chance to be assessed between $85,000 and $115,000. Higher values of COD indicate less uniformity in assessments.

- Price-Related Differential (PRD) is a measure of vertical equity calculated by dividing the mean sales ratio by the weighted mean ratio, where the weight is the sale price. For example, assume a jurisdiction contains two homes, one worth $100,000 assessed at 12% and one worth $1,000,000 assessed at 8% of the fair market value. The mean ratio would be 10% (12% + 8% divided by 2) while the weighted mean ratio would be 8.4% (12% * 100,000 + 8% * 1,000,000 divided by 1,100,000). The resulting PRD (10% divided by 8.4%) would be 1.20. Higher values of PRD indicate greater regressivity.

- Coefficient of Price-Related Bias (PRB) is a regression-based measure that estimates the relationship between the sales ratio and a given proxy for actual property value determined by giving equal weight to market value and assessed value. In other words, PRB predicts the change in assessment ratio that can be expected to result from a 100%

[6] *For a review, see,* Horizontal and Vertical Inequity in Real Property Taxation Author(s): G. Stacy Sirmans, Dean H. Gatzlaff and David A. Macpherson Source: Journal of Real Estate Literature, Vol. 16, No. 2 (2008), pp. 167-180, https://www.jstor.org/stable/44105042.

[7] International Association of Assessing Officers. 2013. *Standard on Ratio Studies*. https://www.iaao.org/media/standards/Standard_on_Ratio_Studies.pdf.

change in this value proxy. For example, a PRB of 0.031 indicates that assessment ratios increase by 3.1% when the home value increases by 100%. Higher values of PRB indicate greater regressivity.

Table 1: IAAO Standards

Parameter	Acceptable Minimum	Acceptable Maximum
COD	5.00	15.00
PRD	0.98	1.03
PRD	-0.05	0.05

The analyses that follow use only "arms-length" transactions, generally meaning only traditional, market-rate sales involving buyers and sellers with no previous relationship (rather than, for example, sales between relatives or foreclosure auctions). For these analyses, we use the assessor's classification of arms-length transactions. In Appendix C, we discuss how the results would change using IAAO definitions.

SUMMARY OF FINDINGS

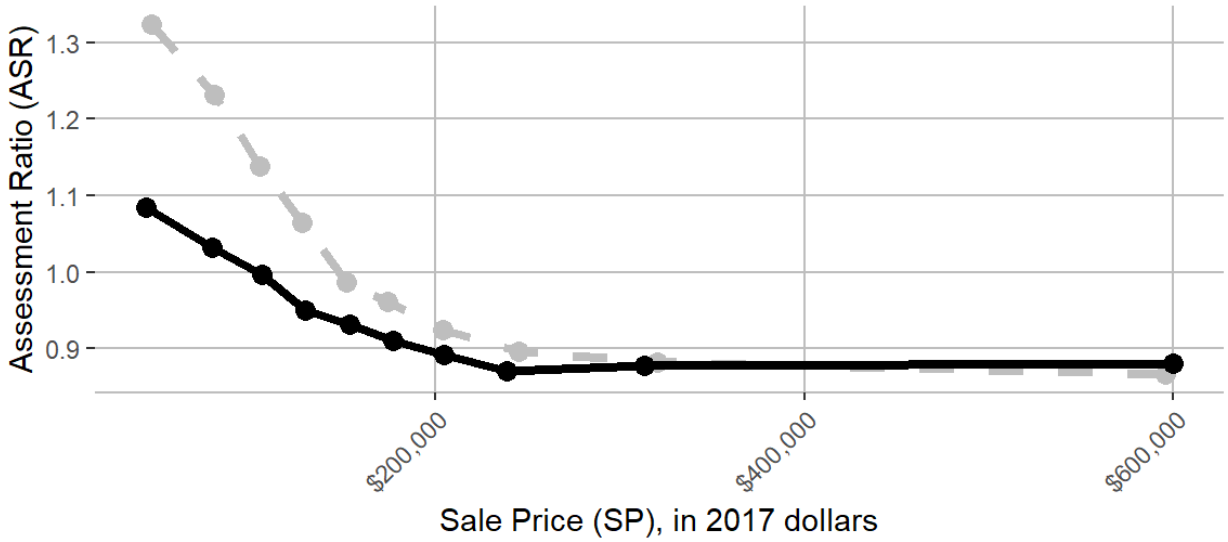
Our evaluation of Philadelphia property assessments reveals moderate regressivity, though various models produced divergent results. While the lowest-valued properties in Philadelphia have, on average, received assessments at roughly 163% of the property's sale price, the city's highest-valued homes are generally assessed at only 87% of the property's sale price. Evaluation using industry-standard measures demonstrate similar levels of regressivity, with more recent years demonstrating acceptable levels or very nearly so.

Sales Ratio Evaluation

The relationship between assessments and sale prices is regressive if less-valuable homes are assessed at higher rates (relative to the value of the home) than more valuable homes. Figure 2 below demonstrates the relationship between assessment ratios and sale prices in Marion County. For Figure 2, property sales have been sorted into deciles (10 bins of equal size based on sale price), each representing 10% of all properties sold in the county. Each dot represents

the average sale price and average sales ratio for each respective decile of properties sold. Figure 2 also compares the most recent values for 2017 (solid line) with the average values across all years of observation, 2013 through 2017 (dashed line). All values were adjusted for inflation to 2017 dollars to facilitate comparisons. If sale prices are a fair indication of market value and assessments are fair and accurate, Figure 2 would be a flat line with a constant sales ratio, meaning that the value of is unrelated to the accuracy of its assessments. A downward sloping line indicates that less expensive homes are over-assessed compared to more expensive homes and is evidence of regressivity.

Figure 2: Assessment Ratio by Sales Price



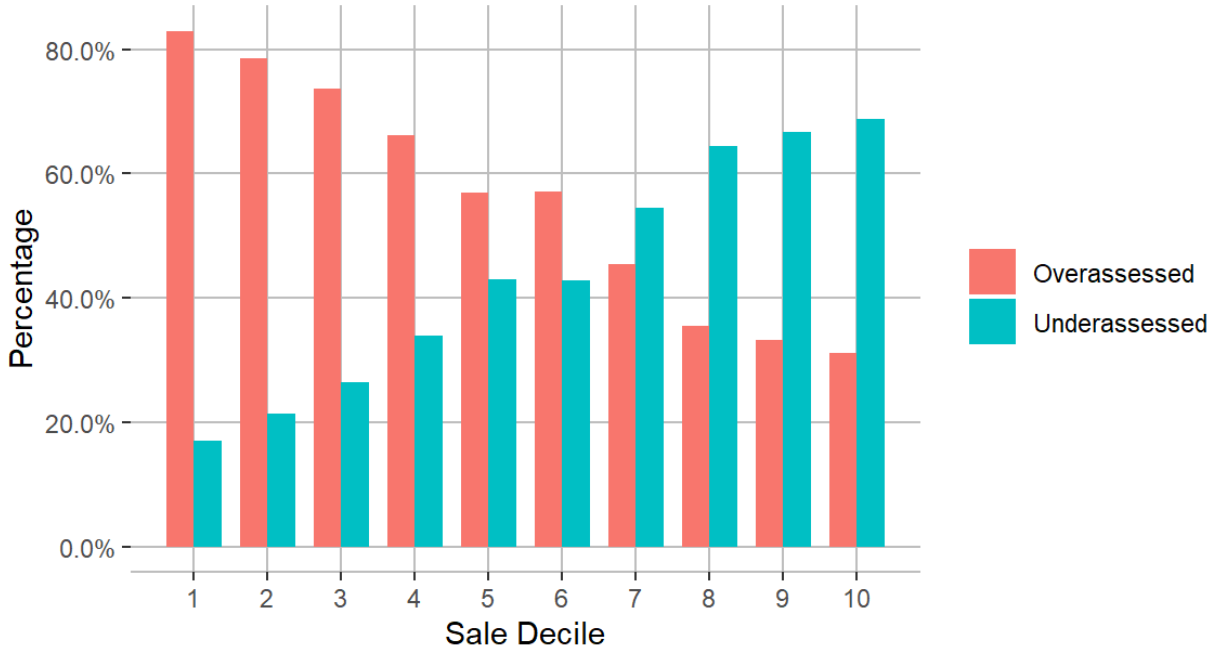
For 2017, the highest ten percent of sales were assessed at 81.2% of the rate of assessment applied to the lowest ten percent of sales. Top decile rate: 88.1%. Bottom decile rate: 108.4%.

As Figure 2 demonstrates, historically, Philadelphia's lowest-valued homes have received assessments nearly one and a half times the assessments received by the community's highest-valued homes, relative to their respective sale prices. By 2017, the final year of observation, this spread had been reduced, from approximately 45 percentage points to approximately 20 percentage points, indicating a significant reduction in overall regressivity.

Figure 3 demonstrates the relative proportion of each decile which was over- or under-assessed. In Philadelphia, assessed values are supposed to be equal to sale price; to that end,

properties are considered “over-assessed” when their assessed value exceeds their market value, while properties are considered “under-assessed” when their assessed value is less than their market value. As Figure 3 shows, some homes in each decile were both over- and under-assessed in any given year. However, the relative proportion of homes that are over- or under-assessed varies significantly based on the value of the property in question. While more than 80% of Philadelphia’s lowest-priced homes received over-assessments, less than 20% of similarly priced homes benefited from under-assessment. Conversely, more than 60% of the city’s highest-priced homes enjoyed underassessments while only approximately 30% of similarly priced homes received overassessments.

Figure 3: Percent of Property Over-/ Under-Assessed by Decile



Industry Standards

The preceding section provides graphical evidence of regressivity in property assessments but it does not provide a statistical evaluation. In this section, we report several standard statistics used in the evaluation of assessment quality.

The International Association of Assessing Officers (IAAO) provides standards for assessments including standards for uniformity and regressivity (*aka* vertical equity). *Uniformity* refers to the overall level of variability in sales ratios across properties. Regressivity refers to the correlation between sales ratios and sale prices. The three main standards are[8]:

- Coefficient of Dispersion (COD) is a measure of uniformity based on the average percentage deviation of the ratios from the median, expressed as a percentage of the median. For example, given a COD of 15%, a property worth $100,000 has a 50% chance to be assessed between $85,000 and $115,000. Higher values of COD indicate less uniformity in assessments.

- Price-Related Differential (PRD) is a measure of vertical equity calculated by dividing the mean sales ratio by the weighted mean ratio, where the weight is the sale price. For example, assume a jurisdiction contains two homes, one worth $100,000 assessed at 12% and one worth $1,000,000 assessed at 8% of the fair market value. The mean ratio would be 10% (12% + 8% divided by 2) while the weighted mean ratio would be 8.4% (12% * 100,000 + 8% * 1,000,000 divided by 1,100,000). The resulting PRD (10% divided by 8.4%) would be 1.20. Higher values of PRD indicate greater regressivity.

- Coefficient of Price-Related Bias (PRB) is a regression-based measure that estimates the relationship between the sales ratio and a given proxy for actual property value determined by giving equal weight to market value and assessed value. In other words, PRB predicts the change in assessment ratio that can be expected to result from a 100% change in this value proxy. For example, a PRB of 0.031 indicates that assessment ratios increase by 3.1% when the home value increases by 100%. Higher values of PRB indicate greater regressivity.

[8] International Association of Assessing Officers. 2013. *Standard on Ratio Studies*. https://www.iaao.org/media/standards/Standard_on_Ratio_Studies.pdf.

Table 1: IAAO Standards

Parameter	Acceptable Minimum	Acceptable Maximum
COD	5.00	15.00
PRD	0.98	1.03
PRD	-0.05	0.05

While no jurisdiction can achieve perfect assessments, remaining within industry-acceptable limits, particularly with regard to COD, PRD, and PRB measures, is an important tool in evaluating equity and uniformity. Table 2 below shows the most recent levels in Philadelphia for all three of these measures, compared with industry recommendations.

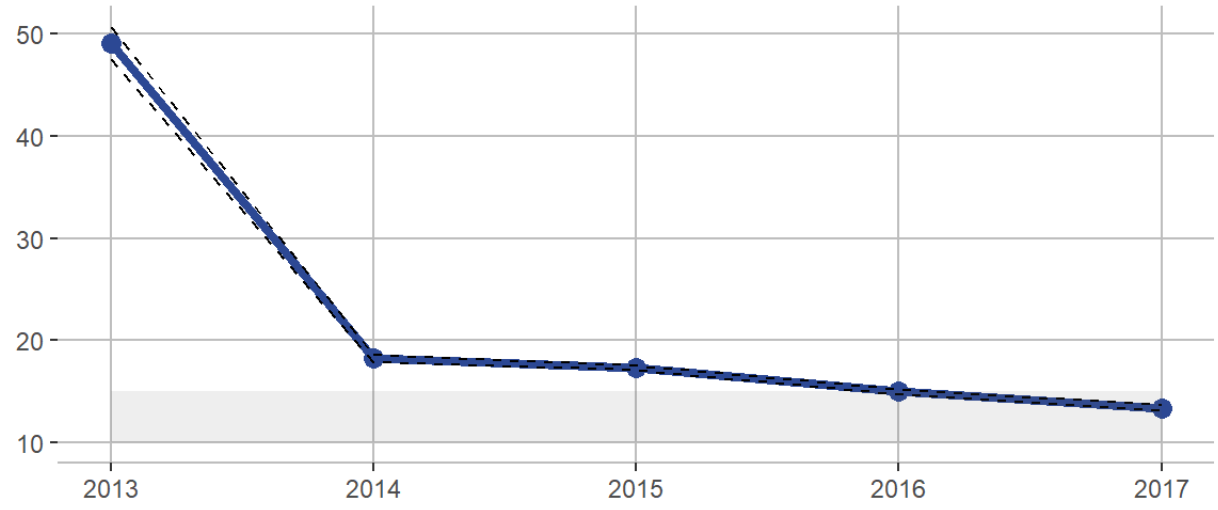
Table 2: Philadelphia's COD, PRD, and PRB Levels (2017)

Measure	Philadelphia Rate	Recommended Limit(s)
Coefficient of Dispersion	13.3770	</= 15
Price-Related Differential	1.038	0.98 to 1.03
Price-Related Bias	-0.0511	-0.05 to 0.05

Philadelphia's COD of 13.37 indicates that while property assessments in the area are not perfectly uniform (an unattainable goal, for practical purposes), the remaining disparities are within normal levels. Similarly, both industry measures of regressivity, the PRD and PRB, are less than 0.01 percentage points beyond acceptable industry thresholds, again indicating that while the system is still not perfect, any remaining inequities of relatively modest.

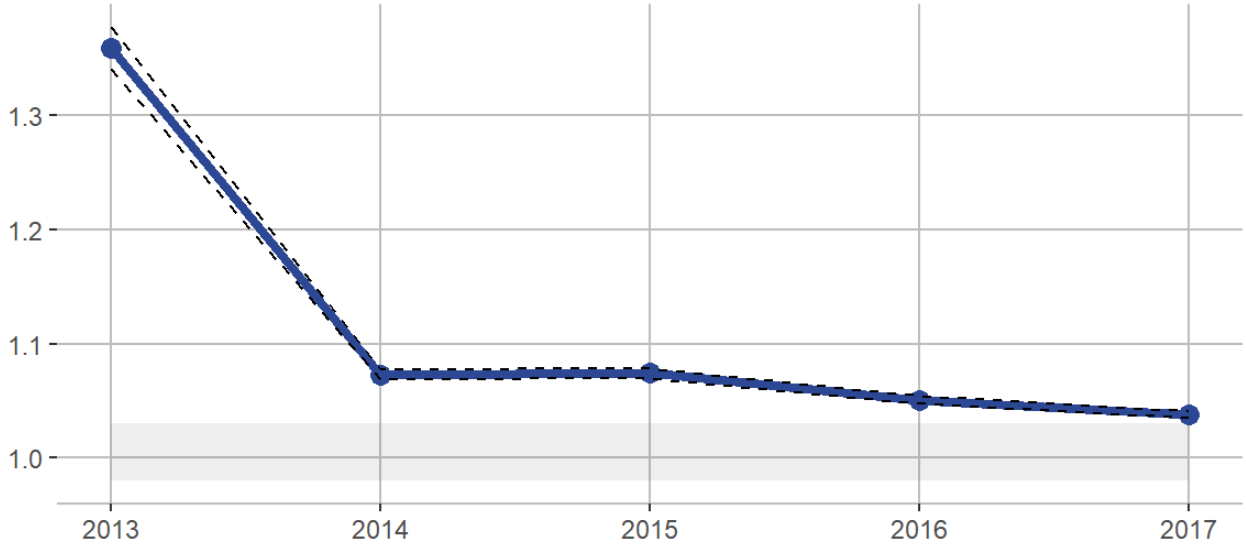
Figures 4 through 6 demonstrate trends over time in industry measures of regressivity and uniformity since 2013. Though all three measures were well beyond industry acceptable limits when our observations began in 2013, all three have since improved substantially. Our evaluation of Philadelphia was significantly limited by data availability prior to 2013. As such, it is unclear whether the initially higher industry measures displayed in Figures 4 through 6 is the result of anomalous conditions in or around 2013, or reflective of a much longer decline.

Figure 4: Philadelphia's Coefficient of Distribution



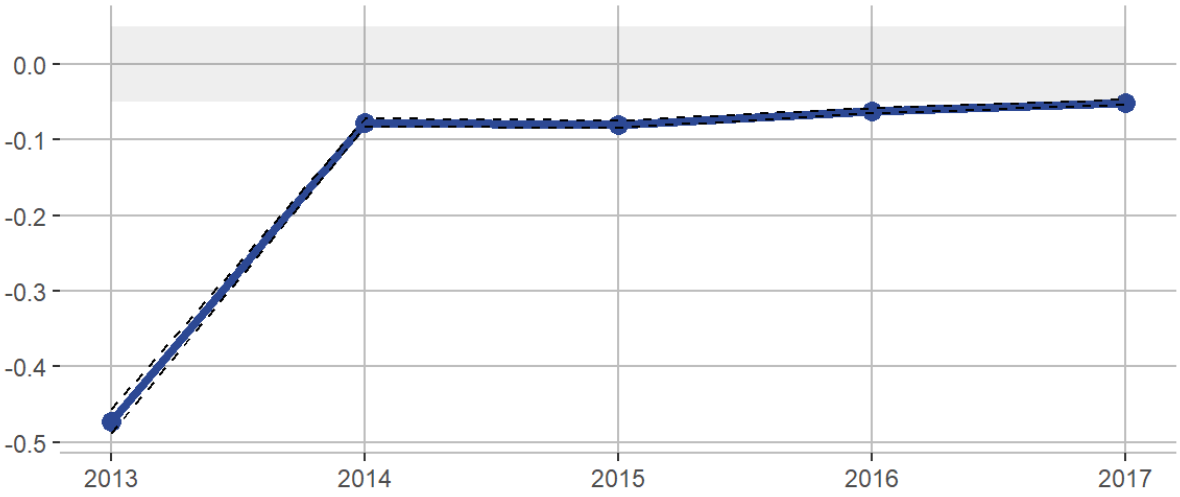
Dotted lines represent 95% Confidence Interval.
In 2017, the Coefficient of Dispersion was 13.38 which does meet the IAAO standard for uniformity.
With this value, a property worth $100,000 has a 50% chance to be assessed between $86620 and $113380.

Figure 5: Philadelphia's Price-Related Differential



Dotted lines represent 95% Confidence Interval.
In 2017, the Price-Related Differential was 1.038 which does not meet the IAAO standard for uniformity.

Figure 6: Philadelphia's Price-Related Bias



Dotted lines represent 95% Confidence Interval.
In 2017, the Price-Related Bias was -0.051 which does not meet the IAAO standard for uniformity.
This value indicates that assessment ratios decrease by 5.1% when assessed value doubles.

Tax Implications

Community Implications

When assessments are regressive, low-value properties can expect to pay more than their fair share of property taxes, while higher-value properties will actually pay less. In other words, regressivity shifts a portion of the collective tax burden from high-value properties and onto lower-value properties. Table 3 provides average sales and assessment data within each decile, including both individual properties and aggregate impact. For example, Line 1 indicates that, on average, among the bottom 10% of homes in Philadelphia, local governments assessed taxes on an additional $3 million worth of property value over the actual value of recently sold properties. By comparison, Line 10 shows that among the county's top 10% of homes, local governments collectively under-assessed recently sold homes by more nearly $60 million in property value. Table 3 supports the findings discussed earlier, namely, that inaccurate assessments in Philadelphia are both inaccurate and regressive, with lower-valued properties being over-assessed, and thus, under-taxed, while higher-valued properties are under-assed, and similarly under-taxed.

Table 3 only uses data from recently sold properties. Scaling the estimates up to all property in Philadelphia requires making some assumptions. Collectively, the under-assessment described in Table 3 amounted to more than $165 million in untaxed property value among recently sold residential properties alone. In an average year, only around 5% of homes in any given community actually sell. As such, the full value of untaxed property is likely many magnitudes greater.

Table 3: Average Sale Price and Total Property Value of Over/ Underassessment Among Recently Sold Homes

Sale Decile	Average Sale	Average Assessed Value	Sum of Sales	Sum of Assessed Values	Sum of Over/Under Assessments	% Over/Under Assessed
1	$45,956	$54,292	$37,113,571	$40,117,200	$3,003,629	7.5%
2	$80,355	$90,846	$67,843,580	$70,009,100	$2,165,520	3.1%
3	$105,097	$111,411	$91,065,333	$90,575,100	-$490,233	-0.5%
4	$127,899	$130,461	$111,383,208	$105,741,200	-$5,642,008	-5.3%
5	$151,655	$147,727	$131,985,768	$122,918,900	-$9,066,868	-7.4%
6	$174,260	$166,181	$152,062,541	$138,492,600	-$13,569,941	-9.8%
7	$203,802	$186,937	$176,254,384	$157,070,200	-$19,184,184	-12.2%
8	$244,994	$217,506	$205,128,351	$178,534,100	-$26,594,251	-14.9%
9	$320,285	$279,213	$269,637,621	$236,480,400	-$33,157,221	-14.0%
10	$595,914	$507,552	$515,071,119	$456,250,900	-$58,820,219	-12.9%

Impact on the Individual Homeowner

A natural question that emerges from our analysis is how much money is at stake for individual homeowners. This question does not have an easy answer because individual property tax burdens can vary even within a single city, as a result of overlapping jurisdictions with concurrent taxing authority. For example, many communities permit municipalities, counties, school districts, public utilities, development districts, and numerous other government entities to levy property taxes. As a result, different residents in the same city or county may be subject to different taxing authorities. For the purposes of the following illustration, we consider the

average 2019 tax rate of 1.3998% calculated by the city, incorporating all various tax rates within the combined city-county. [9]

Table 4 below demonstrates the approximate tax implication for properties within the first, fifth, and tenth deciles of sale prices. Within each decile, we show the average sale price and the average assessed value. We compute the correct tax bill by multiplying the average value by the average tax rate of 1.3998%, and we compare that with the average actual tax bill to arrive at the difference. The difference between the average correct tax bill and the average actual tax bill shows the extent to which the average property in each decile is over- or under-taxed. Consistent with our analysis, these values demonstrate that while the region's lowest-valued homes receive an inflated tax bill, middle- and high-valued homes enjoy increasingly substantial reductions. These estimates should be considered examples rather than definitive conclusions with respect to any individual property because, as noted above, there may be multiple tax rates within a jurisdiction due to different taxing jurisdictions. It should be noted that these figures do not include any exemptions; in reality, most homeowners receive a substantial homeowner exemption that reduces the taxable value of their home.

Table 4: Statutory and Effective Tax Bills Among Property Owners in the city of Philadelphia

Decile	Actual Value	Assessed Value	Statutory Tax Bill	Effective Tax Bill	Difference
Lowest Valued Homes	$43,155.00	$46,780.02	$604.08	$654.83	8%
Median Home Price	$153,472.00	$143,035.90	$2,148.30	$2,002.22	-7%
Highest Valued Homes	$599,617.00	$527,962.77	$8,393.44	$7,390.42	-12%

CONCLUSION

Compared with many other large cities, Philadelphia has a relatively modest level of regressivity among property assessments, even when looking at historic figures. Moreover, the most recent

[9] *Payments, Assistance, & Taxes*, City of Philadelphia (last accessed October 2019), https://www.phila.gov/services/payments-assistance-taxes/property-taxes/real-estate-tax/.

years studied reveal very little regressivity, very nearly within industry-acceptable limits. Nevertheless, the city's lowest-valued homes continue to receive assessments 20 percentage points greater than the community's highest-valued homes, relative to their respective sale prices. Coupled with the more than $250 million in untaxed property value among the city's recently sold homes, these two factors suggest that while Philadelphia may set an example with regard to assessment regressivity, there's is still plenty of room to improve.

Released July 2020

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APPENDIX A

Detailed Findings

A more detailed report including all relevant modeling and results can be found at www.propertytaxproject.uchicago.edu.

APPENDIX B

Results Using IAAO Standards

The analyses in our main report relied on the Marion County assessor's classification of arm's length transaction. In this section, we replicate the analyses using the official IAAO standards for case selection in a sales ratio study.[10] Comparison of local procedures with IAAO procedures in this way is important because, in a small number of communities, local assessors have used a narrower "arms-length" definition to game the system by selecting data points that make the sales ratio study artificially appear better than it is. We do not find evidence of this in Marion County. Indeed, if anything the results generally look less regressive using the IAAO standard rather than the assessor's classification.

Figures 7 through 9 below compare the COD, PRB, and PRD values calculated using the methods outlined above with the values for those same measures computed using the alternative "arms-length" standards recommended by the IAAO. In these graphs, the dashed gray lines represent the values computed using the same arms-length limitations as the local assessor while the blue lines represent these measurements calculated using the IAAO standard. The solid gray area of each graph represents industry-acceptable levels. As these graphs show, using the IAAO standards results in significantly more regressivity among assessments, particularly with regard to COD.

[10] International Association of Assessing Officers. 2013.

Figure 7: Philadelphia's Coefficient of Dispersion

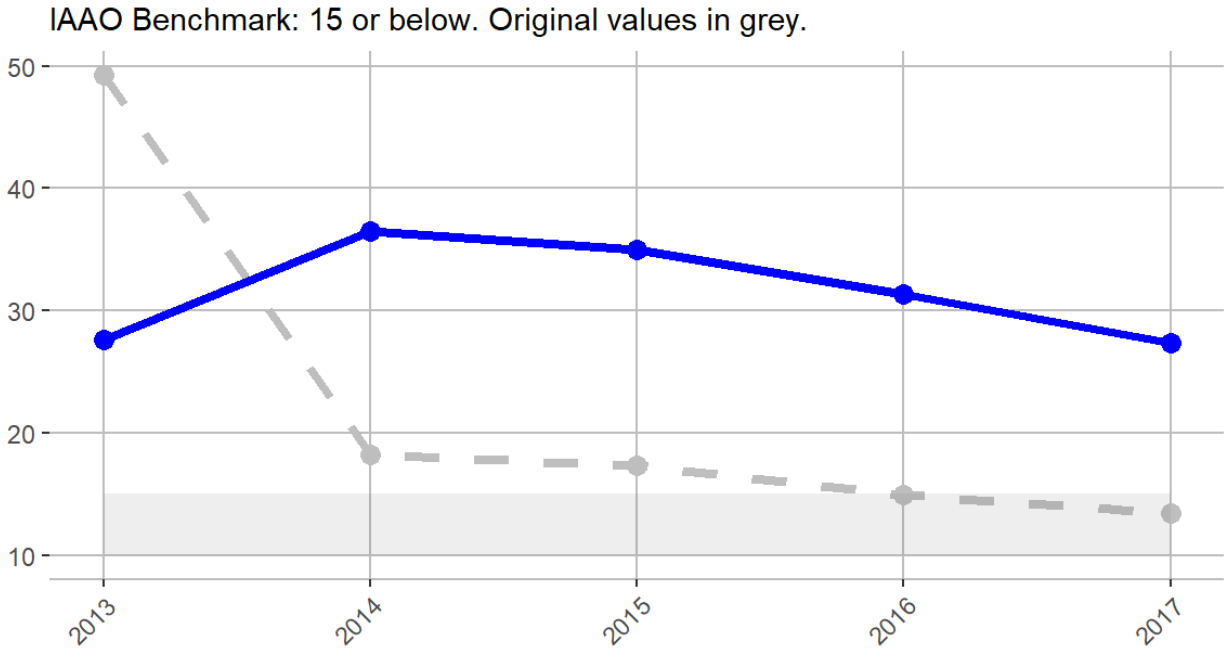


Figure 8: Philadelphia's Price-Related Differential

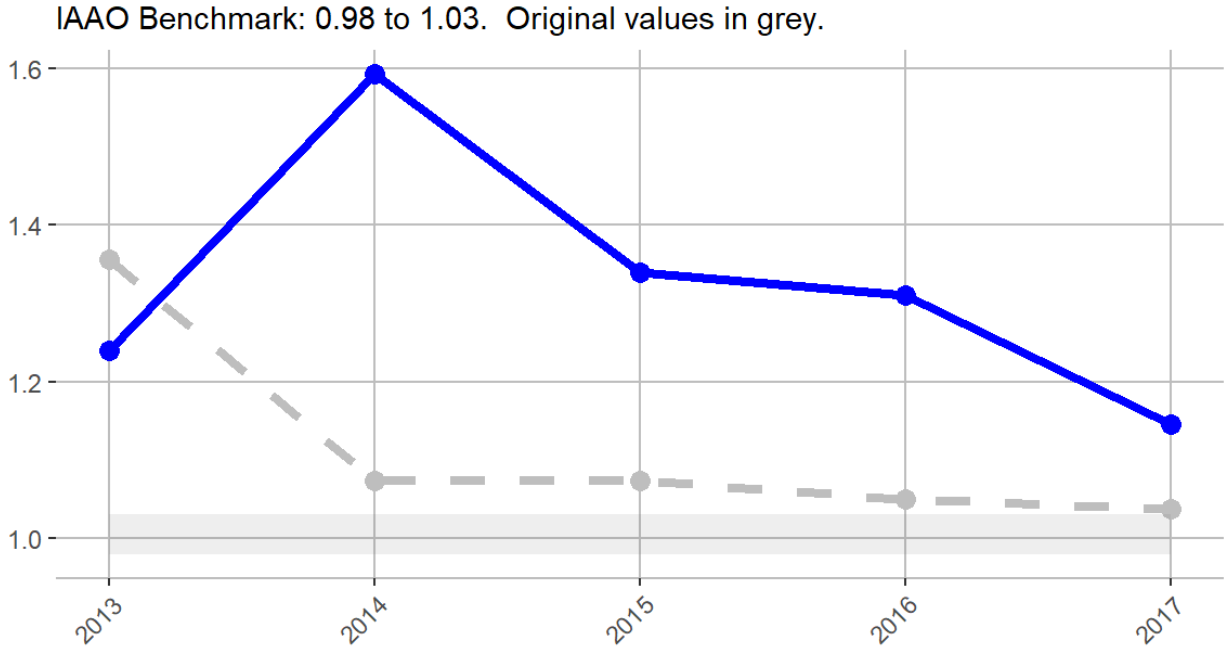
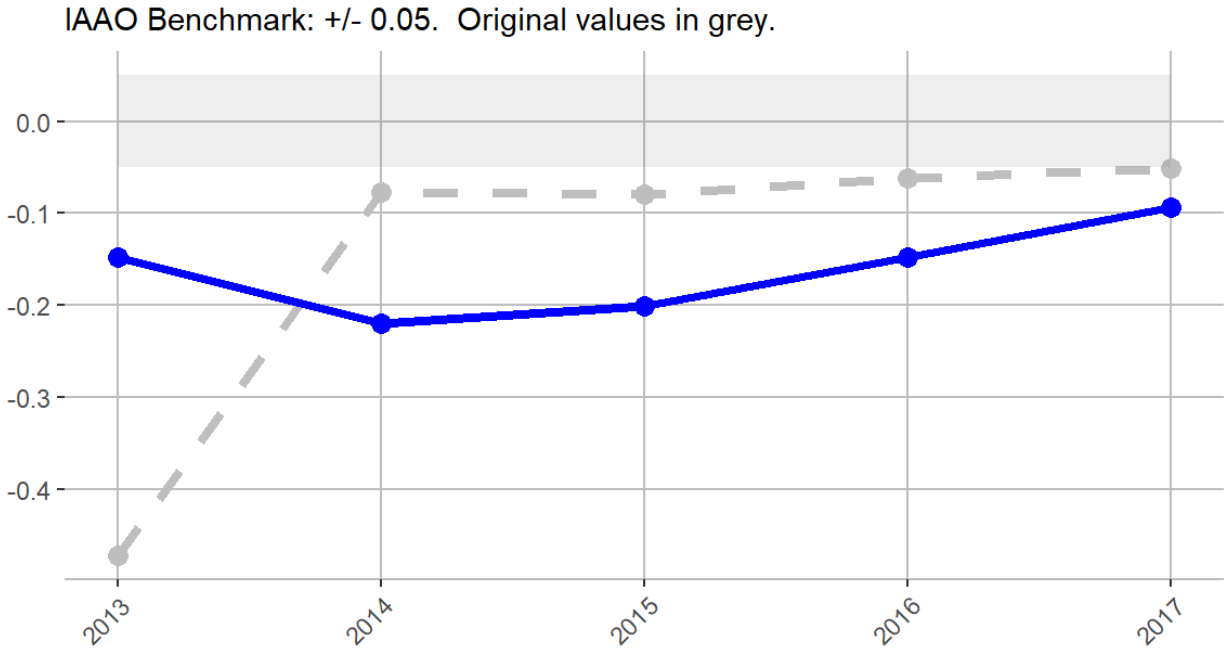


Figure 9: Philadelphia's Price-Related Bias



APPENDIX C

Alternative Measures of Regressivity

While the PRD and PRB measures are the most commonly used metrics within the assessing industry, academic researchers have developed alternative methods with varying degrees of acceptance. Among these alternative models, the majority (6 of 8) produce results similar to those outlined thus far, as Table 5 below shows. See the detailed report in Appendix A for a detailed breakdown of these alternative methods and their results.

Table 5: Alternative Models of Regressivity

Model	Value	Test	T Statistic	Conclusion	Model Description
paglin72	2.4e+04	> 0	79.7	Regressive	AV ~ SP
cheng74	8.0e-01	< 1	520.0	Regressive	ln(AV) ~ ln(SP)
IAAO78	-6.5e-07	< 0	-59.5	Regressive	RATIO ~ SP
kochin82	1.1e+00	< 1	520.0	Progressive	ln(SP) ~ ln(AV)
bell84	2.2e+04	> 0	59.1	Regressive	AV ~ SP + SP^2
	-1.0e-08	< 0	-8.3	Regressive	AV ~ SP + SP^2
sunderman90	-3.1e+04	> 0	-15.0	Progressive	AV ~ SP + low + high + low * SP + high * SP
clapp90	1.2e+00	> 1	449.4	Regressive	ln(SP) ~ ln(AV) -> ln(AV) ~ Z

APPENDIX D

Regressivity Due to Measurement Error

One limitation of sales ratio studies is that a property's sale price may be an imperfect indication of its true market value. Given inevitable random factors in the sale of any individual property, the final price may include some "noise." If so, this will introduce some measurement error into the analysis, which could lead to the appearance of regressivity when there is none. For instance, consider two hypothetical homes that are identical and each worth $100,000. If

both homes went up for sale at the same time, one might fetch a price of $105,000, say if the seller is a particularly savvy negotiator, while the other home might garner only $95,000, say if the buyer is a particularly savvy negotiator. If the assessor appropriately assessed both homes at $100,000, a sales ratio analysis would indicate regressivity (the higher-priced home is under-assessed and the lower-priced home would be over-assessed). While there is no reliable correction for measurement error of this kind, as long as the extent of measurement error is small, relative to the price, the extent of bias will also be small.

We use Monte Carlo simulations to estimate the extent of measurement error that would need to exist for any of our tests to falsely show regressivity due to measurement error or unrelated noise in the data. These tests compare our results with thousands of hypothetical scenarios to determine the likelihood that our same results would be reproduced in the market absent regressivity. As Table 6 shows, these tests demonstrate that for all six measures of regressivity used in our evaluation, home prices would need to vary by more than 25%, or very nearly so, among similar homes to produce the same level of regressivity currently observed in Philadelphia.

Table 6: Monte Carlo Results

Metric	Shock Percentage
COD	24.5%
PRD	>25%
PRB	>25%
Paglin 72	> 25%
Cheng 74	> 25%
IAAO 78	> 25%

APPENDIX E

Comparison with Other Jurisdictions

Table 7: Summary of Communities Included in This Review

Population Rank	Major Metro	Jurisdiction Evaluated	Jurisdiction Population	Revenue from Prop. Tax.	COD	PRD	PRB
1	Los Angeles	Los Angeles County, CA	10,105,518	28.85%	38.75	2.67	0.003
2	Chicago	Cook County, IL	5,180,493	46.26%	16.32	1.04	-0.01
4	Phoenix	Maricopa Count, AZ	4,410,824	28.08%	27.14	0.97	0.21
7	Miami	Miami-Dade County, FL	2,761,581	33.77%	10.8	1	0.01
9	New York*	New York City, NY	8,398,748**	26.27%	58.21	1.07	0.03
12	Seattle	King County, WA	2,233,163	24.26%	10.49	1.01	0.004
13	Las Vegas	Clark County, NV	2,231,647	28.64%	28.35	1.04	0.09
19	Detroit	Detroit, MI	1,753,893	35.99%	70.03	1.71	-0.42
23	Philadelphia	Philadelphia Combined City-County, PA	1,584,138	13.95%	13.41	1.04	-0.05
31	Columbus	Franklin County, OH	1,310,300	34.76%	18.4	1.04	-0.002
32	Minneapolis***	Hennepin County, MN	1,259,428	46.71%	12.91	1.01	0.01
46	St. Louis***	St. Louis & St. Louis County, MO†	996,945	55.37%	17.49	1.08	-0.07
51	Indianapolis***	Marion County, IN	954,670	n/a	22.3	1.06	-0.05
78	Boston***	Boston, MA	807,252††	71.30%	13.15	1.004	0.02

* New York City is coterminous with five counties (New York, Kings, Queens, Bronx, and Richmond) which are all among the nation's most populous. For purposes of this evaluation, these counties were evaluated collectively and are represented in this list by New York.

** This population represents all five counties of New York City, Kings County (Brooklyn) is the actual 9th most-populous county in America with a population of 2,582,830.

*** Though not in the top twenty metros, several other communities were included for various reasons.

†St. Louis and the surrounding county utilize an unusual assessment system between the municipal and county levels, as such both county and city were evaluated. The numbers listed here reflect the entire county.

†† Unlike most large metros which are located near the center of the surrounding county, Boston sits on the border of two counties. As such, this population is unusually small relative to Boston's regional population. When combined with nearby Middlesex County, the regional population is 2,421,966.

APPENDIX F

Glossary

- **Ad Valorem Tax** – A tax applied as a percentage of the value of the item being taxed.
- **Arms-Length Sale** - A sale in the open market between two unrelated parties, each of whom is reasonably knowledgeable of market conditions and under no undue pressure to buy or sell.[11] This generally excludes transfers between family or other close parties, transactions made in a distressed nature, such as through foreclosure or tax sale, and transfers made for substantially little value.
- **Assessment percentage**: The percentage of a property's market value that should be reflected in its assessed value.
- **Coefficient of Dispersion (COD)** - A measure of uniformity based on the average percentage deviation of the ratios from the median, expressed as a percentage of the median. [12]
- **Coefficient of Price-Related Bias** – A regression-based measure that estimates the relationship between the sales ratio and a given proxy for actual property value determined by giving equal weight to market value and assessed value.[13]
- **Price-Related Differential** - A measure of vertical equity calculated by dividing the mean sales ratio by the weighted mean ratio, where the weight is the sale price.[14]
- **Regressivity** – To be characterized as providing an increasing benefit in correlation with an increasing base. When referring to public policies, particularly fiscal policies, this usually reflects a program in which the financial burdens on a given individual decrease as their income or wealth increases.
- **Sales Ratio** – The dollar-for-dollar ratio between a property's assessed value and sale price, where sale price is used as a proxy for market value.[15]

[11] International Association (2013).
[12] *Id.*
[13] *Id.*
[14] *Id.*
[15] *Id.*